Assessing the Quality of Visualizations Produced Using Volume Rendering:
Experiments with a Ray Caster

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Resumo - Um conjunto de parâmetros para avaliação da qualidade objectiva de visualizações, de dados baseados em voxéis, produzidas utilizando um ray caster, é proposto como primeiro passo para a avaliação da qualidade final deste tipo de visualizações. São apresentados resultados obtidos com uma implementação simples de um ray caster a partir de dados sintéticos. O objectivo final desta avaliação consiste no cálculo de “índices de confiança” que facultem ao utilizador aquilo que se poderia chamar uma “visualização guiada” ajudando-o a decidir quais as melhores visualizações de um conjunto de dados.

Abstract - A set of parameters to assess the objective quality of visualizations of a voxel based data set, produced using a ray caster, is proposed as a first step toward the evaluation of the overall quality of these visualizations. Results obtained using synthetic data and a simple implementation of a ray caster are presented. The final goal of this evaluation will be the computation of “confidence indices” that could offer the user a “guided visualization”, i.e. allow him/her to decide what are the best visualizations of a data set.

I. INTRODUCTION

Volume visualization of data has been gaining impact in the last years in several areas including, among others, the visualization of voxel based data produced by medical imaging modalities as CAT, MRI, SPECT or PET [1,2]. A possible approach to volume rendering, becoming more and more popular, is Direct Volume Rendering (DVR), which maps the data in the volume directly into an image (avoiding the intermediate geometric primitives as it is the case with surface fitting) [3].

The quality of a visualisation in general, and of a volume visualization in particular, is a very important issue in the visualization of any kind of data (fundamental in areas as medicine). Assessing it should involve quantifying how “good” are the final images as representations of the data. This quality depends on several factors related to the different “modules” of the visualization process, such as the characteristics of the data acquisition, the pre-processing techniques, the rendering methods and the display. However, in spite of its importance, the issue of quality is often ignored [4], possibly due to the fact that its evaluation is a complex and not “glamorous” task, as put by Nielson [5]. Nevertheless, we believe that an interesting and correct approach to the evaluation of the quality could be the analysis of a number of signal processing issues (such as aliasing and filtering) associated to the production of the images from the data. An introduction to this approach can be found in [6].

As a first step toward quality evaluation of volume visualizations we have identified several processes (modules) along the visualization chain and decided to study some individually, isolating them, as much as possible, from the previous and next modules. Since the volume rendering method is one of the factors that contribute to the overall quality of a volume visualization and it is usually one of the most controllable factors, we have chosen this module to start our study. According to Westover, signal processing is the basis of volume rendering, a fundamental part of the visualization process, since this involves the reconstruction of the input data and a resampling to generate a discrete image, but, unfortunately even sophisticated DVR methods that were developed taking into account the referred signal processing issues (as splatting and Fourier volume rendering [7,8]) are not free of limitations and shortcomings [6]. However interesting it may seem this signal processing approach, even the evaluation of just the contribution of DVR methods to the overall quality of volume visualizations, appears, currently to us, such a complex problem, that we felt we should first study it using an experimental approach to gain some more insight into the involved phenomena [9]. So we have studied a simple “tractable” case, which corresponds to using a nearest neighbour ray caster and a synthetic object. The use of the synthetic object implies a “short-circuit” of the modules previous to the volume rendering, i.e., we are assuming that we have a “perfect” representation of our original volume. To complete the isolation of this module we have performed an analysis, using a set of quality indices, which is independent of any display or human observer since we are assessing the quality directly at the
output of the volume rendering module. In the following sections we will describe in detail these indices, the performed experiments and the obtained results.

We hope that this study, in spite of dealing with a simple case, will pave the way to the more systematic and complete analysis of the problem, needed to attain our final goal: offering the user a guided visualization of his/her data. By this we mean that we aim to be able to compute, for each visualization, a set of parameters that let the user know how “good” is each particular image as a visualization of the original data and help him/her choosing the “best quality” images. These parameters, that we call “confidence indices”, should be as few and as easy to understand as possible.

II. METHODS

Along the process of visualizing voxel based data, several factors may be considered as affecting the final quality of the visualization. Chronologically, the first factor is related to data generation (the characteristics of the data acquisition equipment, i.e. the scanner, and the reconstruction process). Often this is not controllable, and the visualization has to be performed using the available data. The pre-processing techniques, usually applied before the volume rendering method, can also affect the final quality as well as the rendering method itself and the characteristics of the display equipment. Finally, since any visualization is meant to be used by a human observer, this introduces issues, related to visual perception and interpretation of the images, which are much more difficult to evaluate objectively.

As referred, we have decided to start our study by considering a popular volume rendering method, a feedbackwards method [3], an implementation of a ray caster algorithm [10,11]. This ray caster generates a representation of the original scene (existing in a 3D discrete space and voxel composed), on a discrete visualization plane, casting rays into the scene through each element (pixel) of the discrete visualization plane (image). A sampling of the scene is performed along each ray using an adequate sampling rate.

When a sample is taken inside the volume, a contribution to the intensity (grey level or colour) value of the corresponding pixel is computed using the property value of one or several voxels of the volume according to some interpolator. The cumulative contribution of all voxels visited by each ray yields the final image (visualization) of the scene (voxel data set).

In order to isolate the volume rendering process and “short-circuit” the effects of acquisition and pre-processing, synthetic data were used. These choices were made to have a “tractable” case.

To assess the degradation of the overall quality, introduced by the DVR method, we had to estimate some error introduced by the method. Defining error as a difference between the obtained visualization (existing in a 2D discrete space) and the “ideal” visualization (existing in a 2D continuous space), this error is due to several factors involved in computing the final image, such as the finite precision used in the geometrical transformations of the volume and the sampling performed along the rays and on the visualization plane. To estimate the error we had to define the “ideal” visualization (corresponding to the zero error situation); however, this definition is not easy to obtain since we generally don’t know the corresponding image. However in some special cases we can imagine what would be this ideal image (it would exist in a 2D continuous space and be obtained using infinite precision, number of rays and sampling rate along each ray).

A. Ideal experiments

Let us imagine the following experiments:

Experiment 1- consider that our data set is a cube of N x N x N (N even) voxels having the same property value (opacity), a co-ordinate system that can be obtained from the co-ordinate system of the visualization plane by applying integer translations and having equal units (meaning that the voxel and the pixel have the same side), as shown in figure 1.

![Fig. 1 - Image formation in experiment 1](image)

Assume, also, that this data set is a perfect representation of our original volume. If all those conditions are met and a parallel projection is used, the ideal visualization would be a square of side equal to N times the side of a pixel (voxel) and having the same intensity (colour or grey level) (figure 2), which would be proportional to N times the opacity of each voxel. In this situation the ideal image is coincident with the “real” image obtained on the discrete visualization plane and all the voxels of the volume have been sampled exactly once, thus contributing to the image; no voxels were sampled more than once and no voxels were left unsampled. We would say that we have zero error and consequently a visualization of maximum quality.
Experiment 2- now, rotate this cube an angle $\alpha$ around an axis passing through its centre and orthogonal to the visualization plane; the ideal image would still be a square of the same size and intensity, but the real image would no longer coincide with it since, due to above mentioned reasons, some aliasing would result and be noticeable along the edges of the square. In the current situation, some of the voxels of the volume were not sampled and others were sampled more than once (in spite of the fact that along each ray, for each pixel, the same amount of voxels is sampled) consequently the real image is obtained not using the contributions of all the voxels just once, as happened in the previous situation. We would say that an error exists, the quality is no longer maximum, i.e., some degradation has occurred.

Fig. 2 - Image corresponding to experiment 1

Experiment 3- finally, rotate this cube of angles $\alpha$, $\beta$ and $\gamma$, around three orthogonal axis passing through its centre. The ideal image would no longer have uniform intensity, since the ideal rays would not have gone (anymore) through the same distances inside the cube; instead they would have gone through a continuum of distances ranging from zero to the length of the cube’s diagonal. The real image would differ from the ideal one, not only in the edge, but also in the intensity levels that are no longer a continuum, but quantified.

B. How to quantify the error

Considering these situations, how could we estimate the error in order to have a measure of the quality? One possibility would be to count simply the number of voxels not sampled and the number of voxels sampled more than once. The parameters defined this way, would range from zero (no error) to a maximum that depends of the total number of voxels in the volume, i.e. they would vary with the size of the volume and wouldn’t allow error (and hence quality) comparisons among volumes of different sizes. However, dividing them by the total number of voxels, we would obtain volume invariant parameters, which appear much more interesting as quality measures.

If we consider a volume that does not have a uniform property, besides the number of voxels that are not sampled or are sampled more than once, another issue that seems relevant to error estimation is how each voxel contributes to the final image. In the simplest approximation, i.e., using the simplest interpolator (nearest neighbour), a voxel only contributes to the final image if it is sampled itself, however, for more sophisticated interpolators (such as trilinear), a voxel contributes also when certain of its neighbours are sampled. If we think about the simplest case, a possible measure of the error involved in the contribution of each voxel, could be obtained using a distance between the point where the sampling (along the ray) occurs and the centre of the voxel. The rationale behind this measure is the fact that the centre of the voxel is the point where, theoretically, the original volume was sampled and, assuming that the property has a “well behaved” variation inside the voxel, it seems reasonable to suppose that the greater the distance from the centre of the voxel, the greater the difference between the value of the property in the original volume and the sampled and quantified value, i.e., the greater the error. Based on this rationale, we have defined two more quality parameters, using two different distances (Euclidean and City-Block). Each parameter is computed as the mean distance, for all sampled voxels, between the centre of the voxel and the point where the voxel is sampled along a ray. To obtain parameters invariant to the size of the voxel we have divided them by the maximum possible distance (between the centre and a vertex of the voxel). As mean values, they are also invariant to the size of the volume.

In this section we will describe in further detail, the proposed set of quality parameters, the experimental set-up used to collect the information necessary to compute them, as well as the performed experiences.

C. Experimental set-up and collected information

Our ray caster implementation has the following characteristics:

i) orthogonal projection (commonly used in medical applications), which implies parallel rays

ii) sampling performed along each ray with a period equal to the side of the voxel

iii) equal sides of voxel and pixel

To isolate the contribution to the overall quality of the volume rendering method we have used synthetic data as input to this module, which corresponds to consider that all the previous modules of the visualization chain don’t introduce any degradation in the overall quality. The used data is a cube of 16x16x16 voxels.

Inside the ray caster cycle we have collected the following information, for each voxel:

a) number of times that it has been sampled
b) Euclidean and City Block distances, between the centre and the point where it was sampled along the ray, each time it is sampled.

D. Quality indices

As referred, we have used the following error estimates as quality indices:

i) Relative number of voxels not sampled:

\[ N_{ns} = \frac{\text{(n. of voxels not sampled)}}{\text{(n. of samples)}} \]  \hspace{1cm} (1)

ii) Relative number of voxels sampled more than once:

\[ N_{sm} = \frac{\text{(n. of voxels sampled more than once)}}{\text{(total number of samples)}} \]  \hspace{1cm} (2)

iii) Relative mean error, over all samples and using Euclidean distance:

\[ ME_e = \frac{\text{(Sum (Euclidean distance))}}{\text{(maximum distance)}} \times \frac{1}{\text{(total number of samples)}} \]  \hspace{1cm} (3)

iv) Relative mean error, over all samples and using City-Block distance:

\[ ME_c = \frac{\text{(Sum (City-Block distance))}}{\text{(maximum distance)}} \times \frac{1}{\text{(total number of samples)}} \]  \hspace{1cm} (4)

E. Performed experiments

Since our final goal is to offer the user one or more "confidence indices", as we have called them, that would help him/her to choose the best visualizations, we had to investigate which (if any) of the proposed quality indices were adequate. Considering that usually the user has at least the possibility to choose the view used to observe the object, we started the study considering the behaviour of the parameters when the object is rotated around itself as if the user was observing it from different viewing directions. Two sets of experiments were performed, one corresponding to the case of rotation on a plane parallel to the visualization plane (as described in ideal experiment 2) and another corresponding to a more general case (as in ideal experiment 3).

The geometrical analysis of the first case allowed an estimation of the smallest rotation angle that resulted in \( N_{ns} \) and \( N_{sm} \) different from zero, as well as an upper bound to these values, which were experimentally confirmed. Using figure 3 it is easy to understand how the former value was found. It shows one quarter of the projection on the visualization plane of a volume containing 16x16x16 voxels (O is the projection of the centre of the volume). Considering that the cube rotates around an axis parallel to zz and passing through its centre, angle \( \alpha \) is a reasonable approximation to the greater angle of rotation which still implies that all voxels are sampled. Since the co-ordinates of P and Q, considering O as the origin, are respectively \((N/2,N/2-1)\) and \((N/2-0.5,N/2-0.5)\) for a volume of NxN voxels along xx and yy, we have:

\[ \alpha = \arccos \left( \frac{D_1}{D_2} \right)^{1/2} \]  \hspace{1cm} (5)

with \( D_2 = \frac{(N/2)^2 + (N/2-1)^2}{2} \) and \( D_1 = \frac{(N/2-0.5)^2 + (N/2-0.5)^2}{2} \)

Fig. 3 - Calculating \( \alpha \)

To calculate the maximum possible value of \( N_{ns} \) and \( N_{sm} \) we can use figure 4, that shows the geometrical configuration corresponding to the situation where the number of voxels not sampled is maximum; this implies that, for each non-sampled voxel, four neighbours are necessarily sampled. Arranging the voxels according to this rule and trying to "pack" as many non sampled voxels as possible, we were able to establish the theoretical maximum bound for these parameters as:

\[ N_{ns}^{\max} = N_{sm}^{\max} < 0.25 \]  \hspace{1cm} (6)

Fig. 4 - Calculating the maximum values of \( N_{ns} \) and \( N_{sm} \)

Since this seems to correspond to the best possible case, we expect slightly smaller values for our volume.

In order to confirm these findings we have computed the variation of \( N_{ns} \) and \( N_{sm} \) for the described rotation of \( \alpha \) ranging from 0º to 360º, which also allowed to find the angle corresponding to the maximum values of \( N_{ns} \) and \( N_{sm} \). For this angle we have generated the final image of the volume and images containing the projection, of voxels not sampled and sampled more than once, on a face of the cube. The plots and images were obtained using MATLAB [12]. Since we have proposed other two parameters for the case of nearest neighbour interpolator (\( ME_e \) and \( ME_c \)), we have also computed their variation along the same range of rotation (0º to 360º). We expected
to be able to evaluate how discriminating were the proposed parameters so that we could choose which of them should be given to the user as quality parameters. Some of the results we obtained are described in the next section.

Finally, concerning the case of rotations around three orthogonal axis, described in ideal experiment 3, we would like to generalise the previous results and we are currently working in that direction, using the same type of approach. However, we have just computed and visualised the variation of the parameters, hoping to get some more understanding that will lead us into that generalisation.

### III. RESULTS

The plots of the computed values of \(N_{ns}, N_{sm}, ME_e\) and \(ME_c\) corresponding to experiment 2, are shown in figures 5 and 6. We have investigated the variation of these parameters with several resolutions, from 1° to 1/16°, in order to assess if the previewed results and general features were maintained, as this was the case we show the plots obtained using a 1° resolution. Observing these plots, the most notorious feature is their periodicity and symmetry.

However, the fact that the variation of all the parameters, with the rotation angle \(\alpha\), has a period of 90° and is symmetrical around 45° wasn’t a surprise since this result could also have been anticipated through the geometrical analysis of the problem. Other possible observations are:

i) \(N_{ns}\) and \(N_{sm}\) seem very similar

ii) \(ME_e\) and \(ME_c\) seem as measures of the same error (the variation is the same, except that \(ME_c\) have greater values) and are less discriminant than \(N_{ns}\) and \(N_{sm}\)

iii) all the parameters have minimum values equal to zero that occur at \(\alpha = 0°\) and multiples of 90°; there are also other values of \(\alpha\) that correspond to \(N_{ns}=N_{sm}=0\). For the volume of 16x16x16 voxels, we found: \(\alpha = 3.7°\) as the smallest value of \(\alpha\) that corresponds no longer to \(N_{ns}=N_{sm}=0\) and \(N_{ns_{\text{max}}}=0.234\) as the maximum value of these parameters. These findings seem to agree with the values obtained using expressions (5) and (6), namely \(\alpha < 3.8°\) and \(N_{ns_{\text{max}}}< 0.25\).

iv) \(N_{ns}\) and \(N_{sm}\) have maximum values, near to the predicted value, for \(\alpha = 28° + k.90°\) and \(\alpha = 63° + k.90°\), (with \(k=0,1,2,...\)).

Taking this value of \(\alpha = 28°\) for experiment 2, we have visualized the final image of our cube, as well as the projection of the voxels non sampled and sampled more than once on the face of the same cube. Figure 7 a) shows the obtained final image of the cube, the result of our ray caster, which has, as expected, constant grey level and some aliasing noticeable along the edges. In figures 7 b) and c) we can observe a pattern of non sampled voxels and voxels sampled more than once, confirming the
geometrical analysis and reasoning that led us to the maximum value of Nns and Nsm.

In order to evaluate if MEe and MEc are, in fact, basically measuring the same error (as stated in ii) we have also made some more tests and visualizations, which seem to confirm that idea; however for the sake of brevity we do not include their results.

To verify if expressions (5) and (6) also apply to other cubic volumes (of other sizes) we have used a volume of 8x8x8 voxels, and obtained values that can be considered as confirming the predicted ones (α=8º and Nnsmax=0.218, for the smaller angle that no longer yields Nns=Nsm=0 and the maximum value of these parameters, respectively).

Finally, we have made the same kind of experiments using a parallelepiped as synthetic volume, and confirmed the expected results using the same type of analysis.

IV. CONCLUSIONS

We have developed a method for the analysis and estimation of the error introduced by a ray caster (a widely used volume rendering method), which we consider an important contribution to the degradation of the overall quality of a volume visualization obtained using this method. A set of parameters used as error estimations was tested as potential quality indices, in a simple and controlled situation. Two of these parameters, the relative number of voxels not sampled and sampled more than once, seem rather discriminant and general. It is our opinion that they convey an information relevant to any type of ray caster since, whatever interpolator used, the fact that some voxels are not sampled by any ray and others are sampled more than once seems a fundamental issue in the introduced error and consequently in the final quality of the visualization. The other two parameters, due to the fact that they were defined specifically for the nearest neighbour case and seem less discriminant, appear less interesting as the potential quality indices we are looking for.

Considering all the experiments we have performed, the main outcome is, in our opinion, the existence of privileged observation directions (corresponding to a smaller error and hence better quality) that should be used and others that should be avoided, whenever possible. This outcome is based on a set of experiences considering just a rotation around one axis, however the preliminary results we already have concerning rotations around three orthogonal axis (a more general case) seem to reinforce it. The last step toward the generalisation of this result will correspond to the application of the proposed method of error estimation (and hence quality evaluation) to real data, which usually will not be a cube neither a parallelepiped. In that (more) general case we foresee two possibilities:

i) the “brute force” approach we have been using in our experimental approach (i.e., collecting the needed information inside the ray caster cycle) which, in most cases, will probably imply not affordable time and memory costs;

ii) the use of the parameters computed for the extent box of the volume (a parallelepiped) as an estimation of the maximum error (an information considered in many engineering problems as very useful and often satisfying).

The first approach appears adequate to assess the efficacy of the second one, which will have the great advantage of being more efficient. This efficiency will be due to the fact that the same table (containing the error estimated for a reasonable set of observation directions) could be used for all the visualizations of a specific volume. Moreover, this table could be generated before the first visualization and the consumed time would account as an overhead.

To conclude, we would like to stress that this is just an exploratory and introductory study of a subject we
consider fundamental (and usually over looked) in scientific data visualization in general and even more fundamental in the case of medical data. We expect that, putting more effort in the pursuing of this approach, we eventually will be able to compute what we have called "confidence indices" for all the images generated as a visualization of any data set. Of course we cannot forget that the analysis we have made and the parameters we have tested correspond only to a part of the visualization process, the volume rendering, and an effort must also be done to analyse the rest of the process. When this goal is achieved we will be able to provide guidance to the user, along the process of analysing (through visualization) his/her data, with respect to the quality of the visualization. Should anyone succeed in doing the entire work and, we believe, this would be an important reason to make users more confident in Visualization.

REFERENCES


ACKNOWLEDGEMENT

The authors wish to thank José Luis Azevedo and José Vieira, for their help which has made easier the implementation of the experiments.